G-equivariant Hodge atoms (and symbols)

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Notation and recap

Throughout:

- $X/\mathbb{C}$: smooth and projective variety. n will generically stand for its dimension.
- G will be a finite group, usually $\mathbb{Z}/m$
- if G acts regularly and generically freely on X we say that X is a G-variety
- two G-varieties $X,\ Y$ are said to be G-equivariantly birationally equivalent if there exists a G-equivariant birational map between them
- a *linear G-action* on $\mathbb{P}^n$ is the projectivization of a linear representation $G \to \mathrm{Gl}(V),\ \mathbb{P}^n = \mathbb{P}(V)$
- a G-variety X (shortly (X, G)) is said to be *non-G-linearizable* if X is not G-birationally equivalent to $\mathbb{P}^n$ with a linear G-action.

Some known obstructions to linearizability

- existence of fixed points upon restriction to abelian groups
- let $p \in X$ be a fixed point by G and let $\{a_1, \ldots, a_n\}$ be characters for the G-action in T_pX. Set $\det(a_1 \ldots a_n) := a_1 \wedge \ldots \wedge a_n \in \Lambda^n G^\vee$[1]. This quantity is a G-equivariant birational invariant.
- the theory of *modular symbols* due to Kontsevich–Pestun–Tschinkel and its descendents (we will come back to that later)

Today's Goal

Construct new invariants combining G-**equivariant Hodge atoms** with contributions from the geometric fixed locus.

[1] $G^\vee := \mathrm{Hom}(G, \mathbb{C}^\times)$

First results

Recall that any smooth hypersurface $X(1,1,1,1)$ of multi-degree $(1,1,1,1)$ in $(\mathbb{P}^1)^4$ is rational. The projection to the first three factors identifies it with the blowup of $(\mathbb{P}^1)^3$ in an elliptic curve.

Theorem (C–Katzarkov–Kontsevich)

Assume that $X(1,1,1,1)$ is invariant under the $\mathbb{Z}/2$-action induced by swapping the first and second $\mathbb{P}^1$ factors and the third and fourth $\mathbb{P}^1$ factors. Then $(X(1,1,1,1), \mathbb{Z}/2)$ is non-linearizable.

Corollary (C–Katzarkov–Kontsevich)

For any subgroup $G < \mathcal{S}_4$-action on $X(1,1,1,1)$ induced from a transitive action in $(\mathbb{P}^1)^4$ either

(a) *the G-action on the set $\{1,2,3,4\}$ has a fixed point, and in this case $X(1,1,1,1)$ is G-birationally equivalent to $\mathbb{P}^3$ with a G-action*

(b) *the G-action on the set $\{1,2,3,4\}$ has no fixed points, and in this case $(X(1,1,1,1), G)$ is non-linearizable.*

The Non-Archimedean A-model F-bundle

Input: Gromov-Witten theory of X.

- Moduli stack $\overline{\mathcal{M}}_{0,n}(X,\beta)$ and virtual class $[\dots]_{\mathrm{vir}}$
- **GW Potential:** $\Phi(q;t) \in \mathbb{Q}[[q]][[t]]$.

Analytification: Work over the non-archimedean field $\Bbbk = \overline{\mathbb{Q}}((y^{\mathbb{Q}}))$ (Puiseux series).

- **Base Space (B_X):** The "domain of convergence" for Φ.

$$B_X \subset \underbrace{\mathcal{T}_{\Bbbk}^{\mathrm{an}}}_{\text{Kähler/NS}} \times \underbrace{(\mathrm{H} \otimes \Bbbk)^{\mathrm{an}}}_{\text{Cohomology}}$$

- **F-Bundle** $(\mathcal{H}, \nabla)$**:** A flat bundle over $B_X \times \mathbb{D}_u$.

The Connection (Quantum Product)

The connection ∇ encodes the quantum product $\star$:

$$\nabla_{u\partial_u} = u\partial_u + u^{-1}(\mathrm{Eu}\star) + \text{grading terms}$$

where $\mathrm{Eu} = c_1(TX) + \dots$ is the Euler vector field.

Symmetry: The Mumford-Tate Group

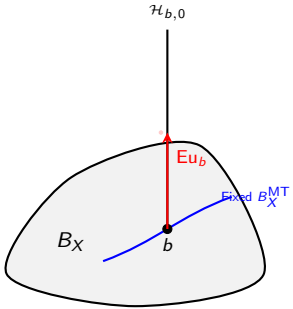
The Group MT

- $\mathrm{Rep}(\mathsf{MT}) \cong \mathcal{HS}^{\mathbb{Z}/2}(\mathbb{Q})$
- It acts on $\mathrm{H}^*(X, \mathbb{Q})$ preserving Hodge classes.

Action on the F-Manifold B_X

- GW invariants arise from algebraic cycles $\implies$ they are MT-invariant.
- MT acts on the base B_X (trivially on $\mathrm{NS}(X)$).
- We restrict attention to the fixed locus:

$$B_X^{\mathsf{MT}} \subset B_X$$



Defining Local Hodge Atoms

Consider the operator $\boldsymbol{\kappa} := \nabla_{u^2\partial_u}\Big|_{u=0} = \mathrm{Eu} \star (-)$ restricted to the fixed locus B_X^{MT}.

The Spectral Cover $\widetilde{U}_X$:

- Look at the eigenvalues of $\boldsymbol{\kappa}$ as functions over the base B_X^{MT}.
- These eigenvalues form a ramified cover $\widetilde{B} \to B_X^{\mathrm{MT}}$.

Definition (Local Hodge Atoms)

Let $U_X \subset B_X^{\mathrm{MT}}$ be the locus where the spectrum is maximal (generic). A **Local Hodge Atom** α is a connected component of the spectral cover:

$$\alpha \in \pi_0(\widetilde{U}_X), \quad \text{where } \widetilde{U}_X = \widetilde{B}_{\mathrm{red}} \times_{B_X} U_X.$$

Hodge atoms

Definition

The set of Hodge atoms is the quotient

$$\mathrm{HAtoms} \;=\; \left(\bigsqcup_{[X]} \pi_0(\widetilde{U}_X) \Big/ \mathrm{Aut}(X) \right) \Big/ \sim$$

The equivalence relation $\sim$ is generated by isomorphisms of F-bundles:

1. **Disjoint Unions**:
$$\mathrm{Atoms}(X_1 \sqcup X_2) \;\cong\; \mathrm{Atoms}(X_1) \sqcup \mathrm{Atoms}(X_2)$$
2. **Blowups**: If $Z \subset X$ has codimension c:
$$\mathrm{Atoms}(\mathrm{Bl}_Z(X)) \;\cong\; \mathrm{Atoms}(X) \sqcup \underbrace{\mathrm{Atoms}(Z) \sqcup \cdots \sqcup \mathrm{Atoms}(Z)}_{c-1 \text{ times}}$$
3. **Projective Bundles:** If $E \to X$ has rank r:
$$\mathrm{Atoms}(\mathbb{P}(E)) \;\cong\; \underbrace{\mathrm{Atoms}(X) \sqcup \cdots \sqcup \mathrm{Atoms}(X)}_{r \text{ times}}$$

Representations associated to Hodge Atoms

1. **Realization:** Any atom $\boldsymbol{\alpha} \in \mathrm{HAtoms}$ is realized by a local atom U_α in the spectral cover of the operator:

$$\boldsymbol{\kappa} = \mathrm{Eu} \star (-) : \mathcal{H}_0|_{B_X^{\mathrm{MT}}} \to \mathcal{H}_0|_{B_X^{\mathrm{MT}}}$$

   inside the A-model F-bundle of some smooth projective X.

2. **MT-Invariance:** The generalized eigenbundle $\mathcal{E}^\alpha \subset \mathcal{H}_0|_{U_X}$ is invariant under the Mumford-Tate group (MT).

Rigidity Lemma

The fiber $\mathcal{E}_b^\alpha$ comes from a rigid representation $E^{\boldsymbol{\alpha}}$ of MT.

- $E^{\boldsymbol{\alpha}}$ is a finite-dim $\overline{\mathbb{Q}}$-linear representation.
- Its isomorphism class depends **only** on $\boldsymbol{\alpha}$ (independent of X or b).

Resulting Invariants:

- **Dimension:** $\rho_{\boldsymbol{\alpha}} = \dim(E^{\boldsymbol{\alpha}})$.
- **Hodge Polynomial:** $P_{\boldsymbol{\alpha}}(t) \in \mathbb{Z}[t, t^{-1}]$ (graded dimensions of $E^{\boldsymbol{\alpha}}$).

The Main Theorem (KKPY)

Theorem (Katzarkov–Kontsevich–Pantev–Yu)

Let X be smooth projective. At a generic point $s \in B_X^{\mathrm{MT}}$, the cohomology $\mathrm{H}^*(X, \Bbbk)$ decomposes canonically via the spectrum of κ_s:

1. **Decomposition:**
$$\mathrm{H} = \bigoplus_{\lambda_i} \mathrm{H}_{\lambda_i}$$
where H_{λ_i} are A-model nc*-Hodge substructures labeled by eigenvalues.*
2. **Compatibility:** *This decomposition respects the B-model $\mathbb{Z}/2$-folded Hodge structure. Thus, each H_{λ_i} carries a* **representation of the Mumford-Tate group**.
3. **Birational Invariance:** *If $X \sim_{bir} Y$, their collections of MT-representation classes coincide,* **modulo** *those coming from subvarieties of codimension ≥ 2.*

Corollary: Rationality Criterion

If a d-dimensional variety X ($d \geq 2$) contains an atom $\boldsymbol{\alpha}$ (representation $E^{\boldsymbol{\alpha}}$) that does **not** appear in varieties of dimension $\leq d-2$, then:

X is not rational.

G-Equivariant Hodge Atoms

Definition

Let X be a G-variety. We consider local atoms arising from the fixed locus of the larger group $B_X^{G\times \mathrm{MT}}$.

$$\mathrm{HAtoms}^G := \left(\bigsqcup_{[X]} \text{Local } G\text{-atoms of } X \Big/ \mathrm{Aut}(X) \right) \Big/ \sim_G$$

The equivalence $\sim_G$ is generated by G-equivariant operations:

1. **Disjoint Unions:** Component-wise identification.
2. G-**Blowups:** $\mathrm{Bl}_Z(X)$ where Z is a G-**invariant** smooth center ($c \geq 2$).
3. G-**Bundles:** $\mathbb{P}(E)$ where E is a G-**equivariant** vector bundle ($r \geq 2$).

New Invariant: G-Invariant Hodge Dimension

For a G-atom $\boldsymbol{\alpha}$, we track the dimension of G-invariants:

$$\rho_{\boldsymbol{\alpha}}^G := \dim_{\mathbb{k}} \left(\text{Generalized } \boldsymbol{\alpha}\text{-eigenspace of } \boldsymbol{\kappa}\right)^{G\times \mathrm{MT}}$$

Filtration by Dimension

We filter atoms by the minimal dimension of a variety supporting them:

$$\mathrm{HAtoms}^G_{\dim\leq 0} \subset \mathrm{HAtoms}^G_{\dim\leq 1} \subset \cdots \subset \mathrm{HAtoms}^G$$

where $\boldsymbol{\alpha} \in \mathrm{HAtoms}^G_{\dim\leq d}$ if it appears in a G-variety of dim $\leq d$.

Criterion for Non-Linearizability

Let X be a G-variety of dimension $d \geq 2$.

If X contains a G-atom $\boldsymbol{\alpha}$ such that:

$$\boldsymbol{\alpha} \notin \mathrm{HAtoms}^G_{\dim\leq d-2}$$

then X is **not** G-birationally equivalent to $\mathbb{P}^d$ (linear).

Implication for 3-folds: For $d = 3$, we must understand $\mathrm{HAtoms}^G_{\dim\leq 1}$.

Classification of Atoms in Dim ≤ 1

Lemma (Generators of $\mathrm{HAtoms}^G_{\dim \leq 1}$)

Assume all subgroups $H < G$ act linearizably on $\mathbb{P}^1$. The atoms in dimension ≤ 1 arise **only** *from:*

1. **Dimension 0 (Points):** *Orbits G/H (where $H < G$).*
2. **Dimension 1 (Curves):** *Induced spaces $G \times_H C$, where:*
   - *C is a smooth connected curve of genus $\mathrm{g} \geq 1$.*
   - *$H < G$ acts on C.*

Properties

- If $\boldsymbol{\alpha}$ comes from a point or rational curve: $\rho^G = 1$.
- If $\boldsymbol{\alpha}$ comes from a genus $\mathrm{g} \geq 1$ curve: $\rho^G = 2$.

$X(1,1,1,1)$ with $\mathbb{Z}/2$-action

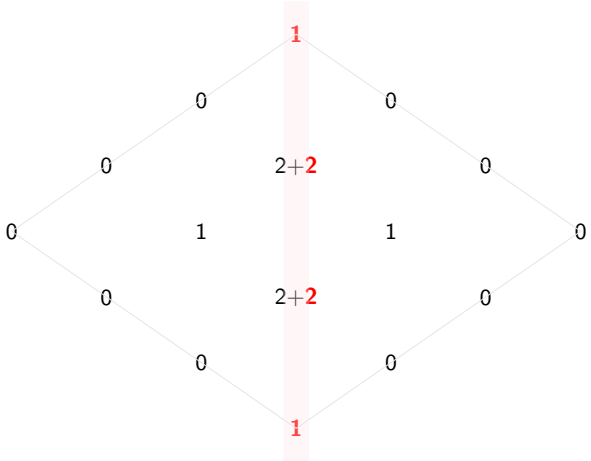
The Variety

Hypersurface in $(\mathbb{P}^1)^4$ of degree $(1,1,1,1)$.

- **Action:** Swap factors $(1 \leftrightarrow 2)$ and $(3 \leftrightarrow 4)$.
- **Invariant Basis:** $h_1 + h_2$, $h_3 + h_4$.

Counting Invariants (ρ^G): Summing the red terms in the diagram:

$$\begin{aligned}\rho^G &= h_G^{0,0} + h_G^{1,1} + h_G^{2,2} + h_G^{3,3} \\ &= 1 + (2) + (2) + 1 \\ &= \mathbf{6}\end{aligned}$$



The Atomic Obstruction for $X := X(1,1,1,1)$

1. Atomic Content of X:

λ	ρ	ρ^G	$P(t)$
16	1	1	1
4	4	2	4
0	5	3	$t+3+t^{-1}$

2. Allowed Atoms (Dim ≤ 1):

Atom	ρ	ρ^G	$P(t)$
Point	1	1	1
2 Pts	2	1	2
Ell. Curve	2	2	$t+2+t^{-1}$

The Arithmetic Contradiction

The $\lambda = 0$ atom has non-trivial cohomology, so it must contain an **Elliptic Curve** atom.

Subtracting the Elliptic Curve:

$$(\rho, \rho^G)_{\text{rem}} = (5,3) - (2,2) = (3,1)$$

The remainder must be a sum of point atoms ($a \times \text{Pt} + b \times 2\text{Pts}$):

$$(3,1) = a(1,1) + b(2,1)$$

$$\implies b = 2, a = -1$$

Conclusion

Negative multiplicity is impossible. $\implies$ X **is not linearizable.**

Symbols

Here, G is *abelian*. Decompose into irreducible components

$$X^G = \sqcup_\alpha F_\alpha.$$

For any $p \in F_\alpha$, the characters $\{a_{i,\alpha}\} \subset G^\vee := \mathrm{Hom}(G, \mathbb{C}^\times)$ for the G-action in T_pX are pointwise independent.

$$\beta(X) := \sum_\alpha [a_{1,\alpha} \dots a_{n,\alpha}].$$

$\mathcal{S}_n(G) := \mathbb{Z}\{[b_1 \dots b_n] : b_i \in G^\vee, \sum_i \mathbb{Z} b_i = G^\vee\}$. Impose the relations

(P) $[b_{\sigma(1)} \dots b_{\sigma(n)}] = [b_1 \dots b_n]$ $\forall \sigma \in \mathcal{S}_n$

(B) for all $b_1, \dots b_n \in G^\vee$

$$\begin{aligned}[b_1 \dots b_n] &= [b_1 - b_2 b_2 b_3 \dots b_n] + [b_1 b_2 - b_1 b_3 \dots b_n] \text{ if } b_1 \neq b_2 \\ &= [b_1 0 b_3 \dots b_n] \text{ if } b_1 = b_2\end{aligned}$$

Theorem (Kontsevich–Pestun–Tschinkel)

The image of $\beta(X)$ in $\mathcal{S}(G)/\sim$ is a G-equivariant birational invariant.

Three "classical" numerical invariants

Theorem (C–Katzarkov–Kontsevich)

Let X be a G-variety of dimension n. The quantity

$$\mathcal{I}(X) = \sum_{F \subset X^G} \mu(F, \mathcal{N}_X F)$$

is a G-birational invariant in the following cases:

- $n = 2$ **($G = \mathbb{Z}/2$):** $\mu(F)$ is $\chi(F)$ for points and $\deg(\mathcal{N})$ for curves.
- $n = 3$ **($G = \mathbb{Z}/2$):**

$$\mu(F) = \begin{cases} 1 & \dim F = 0 \\ 2 - 2\mathrm{g} + \deg(\wedge^2 \mathcal{N}) & \dim F = 1 \\ 4 - 4\mathrm{g} - K_F \cdot \mathcal{N} & \dim F = 2 \end{cases}$$

- $n = 3$ **($G = \mathbb{Z}/3$):** Similar formula involving eigenweights $[++], [+-]$ on normal bundles.

Values of the invariant: Stability and Weakness

Standard Values

The invariant takes constant values for standard actions on projective spaces:

Dim	Group	Value
2	$\mathbb{Z}/2$	4
3	$\mathbb{Z}/3$	6
3	$\mathbb{Z}/2$	8

Example: $X \subset (\mathbb{P}^1)^4$

Let X be the resolution of $x_1x_2x_3 = P(x_4)$, with $\deg P = 3k$.

- **Fixed Locus:** Smooth curve C given by $x^3 = P(x_4)$ with weights [+−].
- **Contribution:** Only from C.

$$\mathcal{I} = 2 - 2g + \deg(\mathcal{N}) = -K_X \cdot C$$

- The value is 6 (same as for $\mathbb{P}^3$), independent of k.

Atomic numerical invariants

Theorem (C.–Katzarkov–Kontsevich)

Let X be a smooth projective threefold with a regular generically free $\mathbb{Z}/p$-action. Fix a type τ (either an integer $g \geq 2$ or a curve C_0). The following quantity is a $\mathbb{Z}/p$-birational invariant:

$$\mathcal{I}_\tau(X) \coloneqq N^\tau_{atoms} - N^\tau_{curves} - 2 \cdot N^\tau_{surfaces}$$

where the terms count components satisfying the conditions below:

	Numerical Case ($\tau = g \in \mathbb{Z}_{\geq 2}$)	**Motivic Case** ($\tau =$ Curve C_0)
Atoms (Trivial action on eigenspace)	Count α with polynomial: $P_\alpha(t) = gt^{-1} + 2 + gt$	Count α with MT-rep: $\rho_\alpha \cong \mathrm{H}^*(C_0)$ ($\bar{\mathbb{Q}}$-linear equiv.)
Curves (Fixed locus)	Count components $F \subset X^{\mathbb{Z}/p}$ with: Genus $g(F) = g$	Count components $F \subset X^{\mathbb{Z}/p}$ with: $\mathrm{Jac}(F)$ isogenous to $\mathrm{Jac}(C_0)$
Surfaces ($S \sim_{bir} C \times \mathbb{P}^1$)	Count surfaces where base C has: Genus $g(C) = g$	Count surfaces where base C has: $\mathrm{Jac}(C)$ isogenous to $\mathrm{Jac}(C_0)$

Application: A Hypersurface in $(\mathbb{P}^1)^4$

The Variety X

Let $P(t)$ be a monic polynomial of degree $3k$ with simple roots.

- **Affine Eq:** $x_1x_2x_3 = P(x_4)$.
- **Compactification:** Embed in $(\mathbb{P}^1)^4$ via charts $x_i = X_i/Y_i$.
- **Hypersurface $\overline{X}$:** Defined by bihomogeneous eq of degree $((1,1,1),3k)$.

The $\mathbb{Z}/3$-Action

Cyclic permutation of the first 3 factors:

$$(x_1, x_2, x_3) \mapsto (x_2, x_3, x_1)$$

(Fixes the 4th coordinate x_4).

The Fixed Locus $X^{\mathbb{Z}/3}$

Locus where $x_1 = x_2 = x_3 = x$.

- **Curve C:** $x^3 = P(t)$ (where $t = x_4$).
- **Cover:** $3:1$ cover of $\mathbb{P}^1_t$, branched at $3k$ roots of P.

Key Properties of C:

1. **Genus:** By Riemann-Hurwitz:

$$2g - 2 = 3(-2) + 3k(3-1)$$

$$\implies g(C) = 3k - 2 \geq 4$$

2. **Normal Weights:** Tangent space splits into a diagonal $\oplus$ normal plane. Action on normal plane has eigenvalues ζ, ζ^2 (weights $[1,2]$).

Application: Calculation and Conclusion

We compute the invariant $\mathcal{I}$ for $\overline{X}$ (using the curve C derived previously).

The Calculation

$$\mathcal{I}(\overline{X}) = \underbrace{\text{Symbolic}(C)}_{\text{Weights } [1,2]} + \underbrace{\text{Atomic}(X)}_{\text{Cohomology}}$$

- **Symbolic:** The pair $(C, [1,2])$ contributes exactly -1.
- **Atomic:** X is rational and $H^{p,q} = 0$ for $p \neq q$. *No atomic contribution.* 0.

$$\Longrightarrow \boxed{\mathcal{I}(\overline{X}) = -1}$$

A surprising rigidity "result"

Theorem (C.–Katzarkov–Kontsevich)

Let X be a smooth projective d-dimensional variety with a G-action. Define the polynomial $P_Y(t) := \sum h^{p,q}(Y)t^{p-q}$.

If there exists $g \in G$ such that:

$$\mathrm{Coeff}_{t^{d-2}}(P_{X^g}) \;>\; \mathrm{Coeff}_{t^{d-2}}(P_X)$$

then (X, G) is **not** *linearizable.*

Example

$$\mathbb{A}^{d+1} \supset X : x_1x_2x_3 = P(x_4, \ldots, x_{d+1})$$

where P is a polynomial of degree $3k$ (with $2k \geq d-1$). Let $\mathbb{Z}/3$ act by cyclically permuting $x_{1,2,3}$. The fixed locus is the hypersurface $x^3 = P(x_4, \ldots, x_{d+1})$. This action is not linearizable.

Chen-Ruan (Stringy) Cohomology

Definition

For a quotient stack $[X/G]$, the Chen-Ruan cohomology is the cohomology of the **Inertia Stack** $\mathcal{I}_{[X/G]}$:

$$\mathcal{I}_{[X/G]} = \{(g, x) \mid gx = x\}/G$$

Explicit Decomposition:

$$H^*_{CR}([X/G]) \cong \bigoplus_{[g] \in \mathrm{Conj}(G)} H^*(X^g)^{C(g)}$$

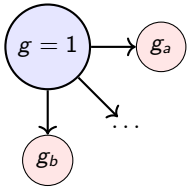
- **Untwisted Sector ($g = 1$):** $H^*(X)^G$.
- **Twisted Sectors ($g \neq 1$):** Cohomology of fixed loci.

Quantum Structure

Gromov-Witten Invariants are defined via moduli of *Orbifold Stable Maps*:

$$\bar{f} : [E/G] \to [X/G]$$

where $E \to C$ is an *admissible G-cover* (balanced nodes).



Inertia Sectors

The F-Bundle and Blowup Conjectures

The A-Model Connection: The quantum product $\star$ on H^*_{CR} defines a Frobenius manifold structure on the base $B_{[X/G]}$.

- **Embedding:** The classical invariant base B_X^G embeds into the stringy base:
$$i_X : B_X^G \hookrightarrow B_{[X/G]}$$
- **Module Structure:** Restricting to $b \in B_X^G$, the twisted sectors $H^*(X^g)$ become **modules** over the algebra $(H^*(X)^G, \star_b)$.

Conjectures: Behavior under Blowups

Let $\widetilde{X} = \mathrm{Bl}_Z(X)$. We expect a canonical isomorphism of F-bundles:

$$\mathrm{Iso} : (\mathcal{H}_{\widetilde{X}}, \nabla) \xrightarrow{\cong} (\mathcal{H}_{X'}, \nabla) \quad \text{where } X' = X \sqcup Z \sqcup \dots$$

1. **Existence:** Holds for quotient stacks $[\widetilde{X}/G] \cong [X'/G]$.
2. **Untwisted Sector:** Maps untwisted to untwisted.
3. **Restriction:** Coincides with the Iritani isomorphism on fixed loci.

New Invariants: Atomic Decomposition of Fixed Loci

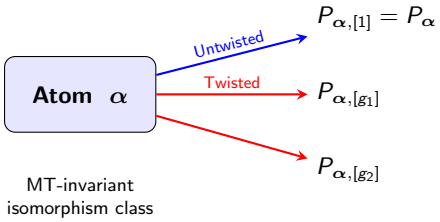
By analyzing the action of the algebra $(H^*(X)^G, \star)$ on the twisted sectors, we refine our atomic invariants.

Definition (Twisted Hodge Polynomials)

For each G-equivariant atom $\boldsymbol{\alpha}$, we obtain a collection of invariants indexed by conjugacy classes $[g]$:

$$\boldsymbol{\alpha} \longmapsto \{P_{\boldsymbol{\alpha},[g]}(t)\}_{[g]\in \mathrm{Conj}(G)}$$

where $P_{\boldsymbol{\alpha},[g]}$ is the Hodge polynomial of the module component $H_{\boldsymbol{\alpha},g} \subset H^*(X^g)$.



Application: A Coefficient Criterion

How do these new invariants constrain the geometry?

Lemma (Dimension Bound)

If an atom α comes from a variety of dimension $\leq d$, then for any g:

$$\mathrm{Coeff}_{t^d}(P_{\alpha,[g]}) \;\leq\; \mathrm{Coeff}_{t^d}(P_{\alpha})$$

Reason: If g acts non-trivially, $\dim(X^g) < \dim(X)$.

Corollary: Obstruction to Linearizability

Let X be a d-dimensional G-variety. If there exists $g \in G$ such that:

$$\mathrm{Coeff}_{t^{d-2}}(P_{X^g}) \;>\; \mathrm{Coeff}_{t^{d-2}}(P_X)$$

then X is **not** G-equivariantly birational to $\mathbb{P}^d$ (linear).

Thank you!