Cubic fourfolds, whole surface contributions, and Hodge atoms

Leonardo F. Cavenaghi

October 2026
Corrected and reorganized seminar notes

Abstract

We explain the mechanism of the September 24 manuscript [1], following the cup pairing, genus-zero operators, integral block cancellation, and the classification of the surviving surface. We give a corrected proof that the operator algebra is generated by primary tensors. We then compare this construction with KKPY's Hodge atoms and Guéré's necessary K3 condition, using the language of non-Archimedean A-model F-bundles. The relative reconstruction and Sarkisov vanishing theorems are used as attributed inputs; the account does not reproduce their full proofs.

1 Part 0: the problem and the two counts

Let $X \subset \mathbb{P}^5_{\mathbb{C}}$ be a smooth cubic fourfold. Its derived category has the semiorthogonal decomposition

$$D^b(X) = \langle \mathrm{Ku}(X), \mathcal{O}_X, \mathcal{O}_X(1), \mathcal{O}_X(2) \rangle.$$

Kuznetsov's conjecture predicts that X is rational if and only if $\mathrm{Ku}(X) \simeq D^b(S)$ for a projective K3 surface S, with no Brauer twist. This is a reasonable prediction: the Serre functor of $\mathrm{Ku}(X)$ is [2], the primitive middle Hodge numbers are $(1, 20, 1)$, and a surface blowup adds a shifted weight-two Hodge structure to middle cohomology. Rational Pfaffian cubics provide familiar examples with a geometric K3 category.

Write $h = c_1(\mathcal{O}_X(1))$ and

$$A_X = H^4(X, \mathbb{Z}) \cap H^{2,2}(X), \qquad T_X = A_X^{\perp}.$$

The lattice $H^4(X, \mathbb{Z})$ has rank 23 and is unimodular. A Hassett label of discriminant d is a saturated rank-two lattice in A_X containing h^2 and having discriminant d. The corresponding divisor is denoted $\mathcal{C}_d$. In this account, *admissible* means

$$d > 6, \quad d \equiv 0, 2 \pmod 6, \quad 4 \nmid d, \quad 9 \nmid d, \quad p \nmid d \text{ for every odd prime } p \equiv 2 \pmod 3.$$

For such d, the geometric K3 condition holds on $\mathcal{C}_d$, as used in [1]. The characterization of all geometric K3 components concerns the union of the admissible divisors, not each fixed divisor separately.

Theorem 1.1 of [1] asserts that, for all sufficiently large admissible d, a very general $X \in \mathcal{C}_d$ is irrational. The threshold is ineffective. Thus the asserted counterexamples concern the sufficiency of the categorical K3 condition.

The test and its sign

Fix a collection $\mathcal{S}$ of Picard-rank-one K3 surfaces of degree d with trivial automorphism group. Define $u(V) = 1$ if the maximal rationally connected quotient of V is birational to a member of $\mathcal{S}$, and $u(V) = 0$ otherwise. The definition depends only on the function field. It satisfies

$$u(\mathbb{P}_Z(E)) = u(Z), \qquad u(Z) = 0 \quad \text{if } \dim Z \leq 1.$$

For a surface Z, $u(Z) = 1$ precisely when Z is birational to a member of $\mathcal{S}$. Birational minimal K3 surfaces are isomorphic.

For a birational map $f : V \cdots\to W$, identify their function fields and let $\mathcal{D}(V), \mathcal{D}(W)$ denote their divisorial valuations. The Lin–Shinder count used in the manuscript is

$$c_u(f) = \sum_{\nu \in \mathcal{D}(W)\setminus\mathcal{D}(V)} u(\kappa(\nu)) - \sum_{\nu \in \mathcal{D}(V)\setminus\mathcal{D}(W)} u(\kappa(\nu)).$$

It is additive under composition. Extraction of a smooth center Z contributes $+u(Z)$; its contraction contributes $-u(Z)$. The argument assumes a parametrization $f : \mathbb{P}^4 \cdots\to X$ and proves

$$1 = c_u(f) = 0. \tag{1}$$

The first equality comes from integral quantum transport. The second comes from the geometry of Sarkisov links. They are separate inputs.

2 Part 1: the cup pairing and the blowup sign

Let M be a smooth projective rationally connected fourfold. Set

$$L_M = H^4(M, \mathbb{Z})/\text{tors}, \quad q_M(a, b) = \int_M a \cup b, \quad T_M = L_M \cap \big(H^4(M, \mathbb{Q})^{\mathrm{Hdg}}\big)^{\perp}.$$

For a smooth projective surface Z, use the same definition in degree two, and write $q_Z(u, v) = \int_Z u \cup v$.

Lemma 2.1 (Hodge-generation)**.** *The forms on $T_{M,\mathbb{Q}}$ and $T_{Z,\mathbb{Q}}$ are nondegenerate. These rational Hodge structures are generated, respectively, by $H^{3,1}(M) \oplus H^{1,3}(M)$ and $H^{2,0}(Z) \oplus H^{0,2}(Z)$. The only off-diagonal types in the even cohomology of M are $(3, 1)$ and $(1, 3)$.*

Proof. Rational connectedness gives $H^0(M, \Omega^p_M) = 0$ for $p > 0$. For example, this follows from very free curves: the pullback of Ω^p_M has negative summands, and such curves cover M. Consequently $H^2(M)$ is of type $(1, 1)$, while hard Lefschetz gives $H^6(M) = L^2H^2(M)$, of type $(3, 3)$. Here L is cup product by an ample class. In degree four, the correct Lefschetz decomposition is

$$H^4(M, \mathbb{Q}) = P^4(M, \mathbb{Q}) \oplus LH^2(M, \mathbb{Q}).$$

The primitive Hodge–Riemann relations and the nondegenerate Lefschetz pairing on the second summand show that the rational Hodge part is nondegenerate. Its orthogonal complement therefore is too. On a surface, the corresponding assertion is the Hodge index theorem.

The transcendental part contains no nonzero rational Hodge classes. By semisimplicity of polarizable rational Hodge structures, a complement to the Hodge substructure generated by the off-diagonal part would be entirely diagonal, hence consist of rational Hodge classes. Such a complement is zero. □

Let $i : Z \hookrightarrow M$ be a smooth surface and $\pi : Y = \mathrm{Bl}_Z M \to M$. Write $j : D \hookrightarrow Y$ for the exceptional divisor and $p : D = \mathbb{P}_Z(N_{Z/M}) \to Z$. Hodge-generation gives

$$i^* T_{M,\mathbb{Q}} = 0, \qquad i_* T_{Z,\mathbb{Q}} = 0.$$

The second assertion uses the codimension-two map

$$i_* : H^2(Z, \mathbb{Q})(-2) \longrightarrow H^6(M, \mathbb{Q}).$$

It sends type $(2,0)$ to $(4,2)$, which vanishes. By contrast, the exceptional-divisor map $j_* p^*$ shifts types by $(1,1)$.

The integral blowup formula and these vanishings give

$$(T_Y, q_Y) = \pi^*(T_M, q_M) \perp (j_* p^* T_Z(-1), -q_Z). \tag{2}$$

Let us check the sign. With $\xi = c_1(\mathcal{O}_D(1))$, one has $j^* j_* \alpha = -\xi\alpha$ and $p_* \xi = 1$. Hence

$$\int_Y j_* p^* u \cup j_* p^* v = -\int_D \xi\, p^*(u \cup v) = -\int_Z u \cup v.$$

The old and exceptional summands are orthogonal by $i^* T_M = 0$. Point and curve blowups add only rational Hodge classes in degree four, so they leave T_M unchanged.

3 Part 2: the operator algebra and primary generation

For numerical curve degree β, consider a connected genus-zero descendant tensor

$$B(a,b) = \left\langle \tau_r(a), \tau_s(b), \prod_{i=1}^{n} \tau_{k_i}(\gamma_i) \right\rangle_{0,\beta}^{M}, \quad a, b \in T_{M,\mathbb{Q}}, \quad \gamma_i \in H^*(M, \mathbb{Q})^{\mathrm{Hdg}}. \tag{3}$$

The ψ classes are cotangent classes on the universal stable-map domain. Nondegeneracy defines A_B by $B(a,b) = q_M(A_B a, b)$. Definition 3.2 of [1] sets

$$G_M = \mathbb{Q}\langle A_B \rangle \subset \mathrm{End}_{\mathbb{Q}}(T_{M,\mathbb{Q}}),$$

as a unital algebra. If $T_M = 0$, use the zero algebra convention. The tensors are rational Hodge tensors; their operators preserve the Hodge structure. The degree-zero tensor $\langle a, b, 1\rangle = q_M(a,b)$ already gives the identity.

Proposition 3.1 (Primary generation; a lemma for these notes)**.** *Let G_M^{prim} be the unital algebra generated by (3) with all descendant exponents zero, allowing arbitrary rational diagonal insertions and all numerical curve degrees. Then*

$$G_M^{\mathrm{prim}} = G_M.$$

Proof. We induct on the total descendant exponent $w = r + s + \sum_i k_i$, simultaneously for all numbers of markings and all degrees. The assertion for $w = 0$ is the definition.

Suppose first that a fixed insertion γ_i carries a positive exponent. Apply genus-zero topological recursion at that marking, using the two variable markings as the reference markings. In each boundary term, both variables lie on one component. The other component has only fixed diagonal

insertions. Pushing its virtual class, with these insertions and its cotangent classes, to the node produces a rational diagonal Hodge class. It is independent of a, b. Thus the component with the variables gives a two-variable tensor with total descendant exponent less than w, and its operator belongs to G_M^{prim}.

We may therefore assume that only the variable markings carry descendants, say $r > 0$. If there are only two markings and $\beta \neq 0$, insert an ample divisor D. The divisor equation multiplies the original tensor by $D \cdot \beta > 0$. Its correction terms vanish because $D \cup T_M = 0$: this is a rational Hodge map to the diagonal group $H^6(M)$ and vanishes on the off-diagonal generators of T_M. We have reduced to at least three markings. Degree-zero two-point unstable terms are not generators; stable degree-zero terms are treated by the same recursion.

Apply topological recursion at the first variable, choosing the second variable and a fixed marking as references. The variables now lie on different components. Test the resulting identity on $a \in H^{3,1}(M)$ and $b \in H^{1,3}(M)$. Conservation of Hodge difference forces the two node inputs along this path to have differences -2 and 2. Those inputs lie in the middle transcendental part. Odd cohomology does not contribute: in a rationally connected fourfold its possible off-diagonal differences are ± 1.

For one splitting, let the two resulting two-variable tensors have operators A_1, A_2, both of lower descendant weight. Contracting the node with the inverse middle pairing gives

$$\sum_{\mu,\nu} q_M(A_1 a, e_\mu) q^{\mu\nu} q_M(A_2 e_\nu, b) = q_M(A_2 A_1 a, b),$$

where $\{e_\mu\}$ is any rational basis of $T_{M,\mathbb{Q}}$ and $(q^{\mu\nu})$ is the inverse pairing matrix. Both operators belong to G_M^{prim} by induction. The identity is an identity of rational Hodge tensors. Its validity on $H^{3,1} \times H^{1,3}$ therefore extends to $T_{M,\mathbb{Q}}$ by Hodge-generation and the nondegenerate pairing of the complementary Hodge types. Summing the stable splittings completes the induction. Swapping the variable markings, when necessary, takes adjoints; the generating family and its algebra are closed under this operation. □

There is no rational basis adapted to each complex Hodge type in this argument. Nor have we assumed that odd cohomology vanishes. For instance, $\mathbb{P}^1$ times a cubic threefold is a rationally connected fourfold with nonzero odd cohomology. These points matter in using the diagonal decomposition in recursion.

Remark 3.2. Proposition 3.1 does not say that divisor insertions alone suffice, or that one small quantum multiplication matrix determines G_M. The primary tensors permit all diagonal inputs. Their generating series are read from the big quantum product and its derivatives; positive-degree two-point terms are recovered using an ample divisor. Descendants remain useful in the relative reconstruction proof, even though they add no generators to this absolute operator algebra.

What is needed from relative reconstruction

Theorem 3.3 of [1] strengthens (2) to

$$G_Y = G_M \times \mathbb{Q}\,\mathrm{id}_{T_Z}, \tag{4}$$

under the classical splitting, with the second factor omitted if $T_Z = 0$. Its proof has three ingredients. Local invariants with two T_Z inputs are scalar multiples of q_Z; mixed tensors vanish; and relative reconstruction identifies the full algebra on the old summand with G_M. The last assertion requires the two reconstruction directions, not just an inclusion.

Here is the geometric reason for the local scalar assertion. If $T_Z \neq 0$, then $p_g(Z) > 0$, so Z is not uniruled. A family of rational curves in a local projective bundle cannot sweep Z in nonzero base degree. Restricting a holomorphic two-form to its image gives zero, and Hodge-generation extends the vanishing to T_Z. In zero base degree, the two variable insertions meet on the same point of Z; fiber integration leaves a scalar multiple of $\int_Z u \cup v$.

Degeneration terms are trees. Along the unique path joining the two variable inputs, contraction is composition of operators, as above; branches carrying only fixed inputs give diagonal Hodge classes. Relative contact data and descendant powers require ordered reconstruction inductions, supplied in Lemmas 3.6–3.7 of [1]. Proposition 3.1 is not a replacement for those inductions.

4 Part 3: the projector and whole integral blocks

Let e be the class of an exceptional fiber $\mathbb{P}^1 \subset D$. Its normal bundle in Y is $\mathcal{O}_{\mathbb{P}^1}(-1) \oplus \mathcal{O}_{\mathbb{P}^1}^{\oplus 2}$, so the degree-one fiber maps have no obstruction from this normal bundle. The two-marked evaluation pushforward gives the fundamental class of $D \times_Z D$. The compactification includes the boundary with a contracted component carrying coincident markings. Therefore

$$\langle j_* p^* u, j_* p^* v \rangle_{0,e}^Y = \int_{D \times_Z D} \xi_1 \xi_2 \, p^*(u \cup v) = q_Z(u, v).$$

There are two minus signs from restricting the insertions, whereas the ambient pairing on the exceptional block is $-q_Z$. The resulting operator is $-P_Z$, where P_Z is the orthogonal projector onto $j_* p^* T_Z(-1)$. On the old transcendental block the tensor is zero. Thus $P_Z \in G_Y$. Together with the reconstruction assertions, this gives the independent scalar factor in (4).

Proposition 4.1 (Whole-block cancellation; [1], Lemma 3.8)**.** *Along a smooth weak factorization starting from $\mathbb{P}^4$, every intermediate fourfold has an integral orthogonal decomposition*

$$T_M = \bigoplus_{\alpha=1}^{m} {}^{\perp} V_{\alpha,\mathbb{Z}}, \qquad G_M = \bigoplus_{\alpha=1}^{m} \mathbb{Q} P_\alpha \simeq \mathbb{Q}^m,$$

where each $V_{\alpha,\mathbb{Z}}$ is a whole shifted surface-transcendental lattice. A surface contraction removes one whole existing block.

Proof. At $\mathbb{P}^4$ the transcendental lattice is zero. A surface extraction adds one integral block and its scalar projector by (2)–(4). Now view a contraction $M \to M'$ as a blowup of its smooth center. The exceptional block $W_{\mathbb{Z}}$ has projector P_W with $P_W G_M = \mathbb{Q} P_W$. Thus P_W is a primitive central idempotent of $G_M \simeq \mathbb{Q}^m$. The primitive idempotents of this algebra are exactly the P_α, so $W_{\mathbb{Q}} = V_{\alpha,\mathbb{Q}}$ for one α. Both integral lattices are direct summands, hence saturated. Consequently

$$W_{\mathbb{Z}} = T_M \cap W_{\mathbb{Q}} = T_M \cap V_{\alpha,\mathbb{Q}} = V_{\alpha,\mathbb{Z}}.$$

This proves integral equality, rather than only a rational identification. □

This argument explains the role of G_M: it distinguishes entire surface contributions even when Hodge theory alone permits additional decompositions. Kulikov's general indecomposability conjecture for integral polarized surface transcendental Hodge structures is false [6]; it cannot replace this operator argument.

One must also distinguish the integral Hodge-isometry class of a block from the isomorphism class of its surface. The former need not determine the latter. Thus whole-block cancellation gives the desired numerical count for the *full* collection of surfaces with the prescribed integral block, not automatically for an arbitrary subcollection.

5 Part 4: rank twenty-one, the test, and special loci

Suppose X is rational. Since $h^{3,1}(X) = 1$, the final decomposition has exactly one nonzero surface block: every nonzero block has a nonzero off-diagonal part. Thus there is a surface Z with

$$(T_X, q_X) \simeq (T_Z(-1), -q_Z), \qquad p_g(Z) = 1, \qquad G_X = \mathbb{Q}\,\mathrm{id}_{T_X}\,.$$

These conclusions, under rationality, do not require a very general point of a Hassett divisor.

Lemma 5.1 (Rank-twenty-one centers; [1], Lemma 3.9)**.** *If $p_g(Z) = 1$ and $\operatorname{rk} T_Z = 21$, then Z is birational to a Picard-rank-one K3 surface S. Its primitive ample class has square $|\operatorname{disc} T_Z|$. If this number is $d > 2$ and $\operatorname{End}_{\mathrm{Hdg}}(T_{Z,\mathbb{Q}}) = \mathbb{Q}$, then $\operatorname{Aut}(S) = \{1\}$.*

Proof. Pass to the minimal model S; p_g and T_Z are unchanged. Since $p_g = 1$, the canonical class is nef and $K_S^2 \geq 0$. Write $q = h^{1,0}(S)$. Noether's formula and the topological Euler characteristic give

$$b_2(S) = 22 - 8q - K_S^2.$$

Projectivity and $\operatorname{rk} T_S = 21$ give $b_2(S) \geq 22$. Thus $q = 0$, $K_S^2 = 0$, $b_2 = 22$, and $\rho(S) = 1$. General type is excluded by $K_S^2 = 0$. A properly elliptic minimal surface has a nonzero isotropic fiber class independent of an ample class, contradicting $\rho = 1$. The classification in Kodaira dimension zero leaves a K3 surface.

Let $N = \mathrm{NS}(S) = \mathbb{Z}L$ and $\Lambda = H^2(S, \mathbb{Z})$. The K3 lattice is unimodular, and N, T_S are primitive orthogonal sublattices. Primitivity of N makes restriction $\Lambda = \Lambda^* \to N^*$ surjective; the same holds with N and T_S interchanged. Hence their discriminant groups are identified by the unimodular gluing, and

$$L^2 = |N^*/N| = |T_S^*/T_S| = |\operatorname{disc} T_Z|.$$

An automorphism fixes the primitive ample generator L. Its action on $T_{S,\mathbb{Q}}$ is scalar by the endomorphism hypothesis, and the pairing forces this scalar to be ± 1. The choice -1 is incompatible with the identity action on the cyclic discriminant group of order $d > 2$. Therefore it acts trivially on $H^2(S, \mathbb{Z})$ and is the identity by the faithful cohomological action for K3 automorphisms. □

For a very general $X \in \mathcal{C}_d$ one has

$$\operatorname{rk} T_X = 21, \qquad |\operatorname{disc} T_X| = d, \qquad \operatorname{End}_{\mathrm{Hdg}}(T_{X,\mathbb{Q}}) = \mathbb{Q}. \tag{5}$$

Choose $\mathcal{S} = \mathcal{S}(T_X)$ to consist of *all* K3 surfaces whose shifted integral polarized transcendental lattice is isometric to (T_X, q_X). Every center with this block is birational to a member of $\mathcal{S}$, by the lemma. Since exactly one such block survives, the additions minus removals give $c_u(f) = 1$.

These hypotheses are sufficient. They are not an "if and only if" description of the locus where the count is one. In particular, $\operatorname{End}_{\mathrm{Hdg}}(T_{X,\mathbb{Q}}) \neq \mathbb{Q}$ does not imply that the associated surface has a nontrivial automorphism. At a higher Noether–Lefschetz point, $\operatorname{rk} T_X$ drops, and the chosen Hassett label d need not equal $|\operatorname{disc} T_X|$. If a surviving smaller-rank block comes from a K3, that K3 has higher Picard rank and is excluded by the rank-one test. These observations do not assert rationality or classify all special points.

The geometric zero

Theorem 6.6 of [1] says that, for sufficiently large d, the same test has $c_u(f) = 0$ for birational maps between the terminal Mori spaces over points occurring here. Its proof factors the map into Sarkisov links. The count of each link is the difference of corrections attached to the two Mori fiber spaces; these corrections telescope and vanish at endpoints over a point.

Uniform bounds for marked link diagrams, surface-link analysis, and control of conic-bundle ramification enter this construction. High degree restricts pencils on Picard-rank-one K3 surfaces: the manuscript uses the inequality $2g - 2 \geq \sqrt{d}$. The uniform threshold remains ineffective. This geometric theorem supplies the second equality in (1); the quantum decomposition alone does not supply it.

6 Comparison with KKPY and Guéré

We now use KKPY's notation [2]. Let $(\mathcal{H}, \nabla)/B_X$ be the maximal non-Archimedean A-model F-bundle, with

$$\mathcal{H}^o = \mathcal{H}|_{u=0}, \qquad \kappa = \mathsf{Eu} \star (-).$$

The group Hod retains Hodge difference $p-q$ and parity, forgetting Tate twists. On B_X^{Hod}, take the locus U_X where the number of distinct eigenvalues of κ is maximal. Components of the reduced spectral cover, modulo the specified elementary equivalences, define the Hodge atoms. Their multiset is the chemical formula $\mathsf{CF}(X)$. The associated representations E^α are generally defined over $\overline{\mathbb{Q}}$, not canonically over $\mathbb{Q}$.

Iritani's decomposition [4, 5] is the input behind

$$\mathsf{CF}(\mathrm{Bl}_Z X) = \mathsf{CF}(X) + (r-1)\mathsf{CF}(Z), \qquad r = \mathrm{codim}_X Z.$$

Thus weak factorization constrains the factors of a rational fourfold by those of points, curves, and surfaces. The full local spectral F-bundle contains more information than its generic Hodge representation. This distinction is essential in Guéré's argument.

Theorem 6.1 (Guéré, [3], Theorem 63)**.** *If X is a rational smooth cubic fourfold, there is a projective K3 surface S and an isomorphism of rational Hodge structures*

$$H^4(X, \mathbb{Q})_{\mathrm{prim}} \simeq H^2(S, \mathbb{Q})(-1).$$

Here is the proof mechanism in F-bundle language. At the normalized small quantum point, the zero spectral factor has fiber

$$V_0 = \left(H^4(X, \mathbb{Q})_{\mathrm{prim}} \otimes \Bbbk\right) \oplus \Bbbk^2, \qquad \dim V_0 = 24.$$

Its residual nilpotent operator N vanishes on the primitive part and has one size-two Jordan block on the ambient part. Guéré's transport argument along blowups forces a surface factor with Hodge-difference-two part of dimension one, no Hodge-difference-one part, and residual nilpotent rank at most one. After passage to a minimal surface, irregular cases are excluded. If its nef canonical class is nonzero numerically, multiplication by that class has rank at least two: it maps H^0 nontrivially into H^2 and some H^2 class nontrivially into H^4. The remaining case is a K3 surface.

The transported $H^*(S)$ factor embeds into V_0. Both have dimension 24, so this is an isomorphism intertwining their residual operators. Taking $\ker N / \operatorname{im} N$ removes the common size-two

Jordan block and yields the claimed middle Hodge structure, with rational descent as in Guéré's proof. One must retain special spectral fibers: V_0 should not be identified without argument with a single generic Hodge atom. The theorem is a rational Hodge-theoretic necessary condition; it does not itself assert integral polarized compatibility or an equivalence of derived categories.

What the operator algebra adds to this description

Proposition 3.1 places G_M within the same genus-zero primary theory that defines the A-model F-bundle. On a factorization from $\mathbb{P}^4$, its primitive central idempotents mark whole surface transcendental contributions. This is the precise atomic interpretation of the algebra in the present argument.

Nevertheless, an algebra in $\mathrm{End}_{\mathbb{Q}}(T_{M,\mathbb{Q}})$ is not literally a KKPY atom. The required transport identifies its distinguished rational subspaces with the *classical integral* blowup summands. The integral lattice then remembers the degree d, which is needed in the geometric count. Iritani's decomposition explains the organizing principle; integral compatibility and the Sarkisov vanishing are substantive further assertions. Consequently the interpretation in terms of atoms is justified, while the claim that nothing further is proved would be stronger than this comparison establishes.

Jérémy Guéré has written a concise account of the argument in the language of atom theory, which will appear soon.[1]

References

[1] OpenAI, *Irrational cubic fourfolds with geometric K3 categories*, September 24, 2026. Public manuscript.

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[1] Announcement based on personal communication reported by the author.